Trigonometric Equations

Question1

If $\tan\left(\frac{\pi}{4}+\alpha\right)=\tan^3\left(\frac{\pi}{4}+\beta\right)$**, then** $\tan(\alpha+\beta)\cot(\alpha-\beta)=$

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

$\sec^2 2\beta+\tan^2 2\beta$

B.

$\operatorname{cosec}^2 2\beta+\cot^2 2\beta$

C.

$2\left(\sec^2 2\beta+\tan^2 2\beta\right)$

D.

$4\left(\sec^2 2\beta+\tan^2 2\beta\right)$

Answer: C

Solution:

$\tan\left(\frac{\pi}{4}+\alpha\right)=\tan^3\left(\frac{\pi}{4}+\beta\right)$

let $x=\frac{\pi}{4}+\alpha, y=\frac{\pi}{4}+\beta$

$\therefore \tan x=\tan^3 y$

Now, $\alpha+\beta=x+y-\frac{\pi}{2}$

and $\alpha-\beta=x-y$

then, $\tan(\alpha+\beta)=\tan\left(x+y-\frac{\pi}{2}\right)$

$= -\cot(x+y)$

and $\cot(\alpha-\beta) = \cot(x-y)$

Thus, $\tan(\alpha+\beta)\cot(\alpha-\beta)$

$= -\cot(x+y)\cdot\cot(x-y)$

and $\cot(x+y)\cdot\cot(x-y)$

$$= \frac{\left(1-\tan^4 y\right)\left(1+\tan^4 y\right)}{\tan^2 y\left(1+\tan^2 y\right)\left(1-\tan^2 y\right)}$$

$$= \frac{1+\tan^4 y}{\tan^2 y} = \cot^2 y + \tan^2 y$$

Thus, $-\cot(\alpha-\beta)\tan(\alpha+\beta)$

$= -\left(\cot^2 y + \tan^2 y\right)$

and $\tan y = \tan\left(\frac{\pi}{4}+\beta\right) = \frac{1+\tan\beta}{1-\tan\beta}$

$\Rightarrow \cot y = \frac{1-\tan\beta}{1+\tan\beta}$

Therefore,

$$\tan^2 y + \cot^2 y = \frac{(1-\tan\beta)^4+(1+\tan\beta)^4}{\left(1-\tan^2\beta\right)^2}$$

$$= \frac{(1-\tan\beta)^4}{(1-\tan\beta)^2(1+\tan\beta)^2} + \frac{(1+\tan\beta)^4}{(1-\tan\beta)^2(1+\tan\beta)^2}$$

$$= \left(\frac{1-\tan\beta}{1+\tan\beta}\right)^2 + \left(\frac{1+\tan\beta}{1-\tan\beta}\right)^2$$

$$= \frac{\left(1+\tan^2\beta-2\tan\beta\right)^2+\left(1+\tan^2\beta+2\tan\beta\right)^2}{\left(1-\tan^2\beta\right)^2}$$

$$= \frac{2\left(1+\tan^4\beta+4\tan^2\beta\right)+2\left(\tan^2\beta+2\tan^3\beta+2\tan\beta+\tan^2\beta-2\tan^3\beta-2\tan\beta\right)}{\left(1-\tan^2\beta\right)^2}$$

$$= 2\left[\frac{1+\tan^4\beta+4\tan^2\beta+2\tan^2\beta}{\left(1-\tan^2\beta\right)^2}\right]$$

$$= 2\left[\frac{\left(\tan^4\beta+2\tan^2\beta+1\right)+(2\tan\beta)^2}{\left(1-\tan^2\beta\right)^2}\right]$$

$$= 2\left(\sec^2 2\beta + \tan^2 2\beta\right)$$

Question2

If $0 \leq x \leq 3$ and $0 \leq y \leq 3$, then the number of solutions (x, y) of the equation $\left(\sqrt{\sin^2 x - \sin x + \frac{1}{2}}\right) 2^{\sec^2 y} = 1$ is

AP EAPCET 2025 - 26th May Morning Shift

Options:

A.

5

B.

2

C.

6

D.

1

Answer: B

Solution:

Let $A = \sqrt{\sin^2 x - \sin x + \frac{1}{2}}$

and $B = 2^{\sec^2 y}$
then, $A \cdot B = 1$
for A to be real

$\sin^2 x - \sin x + 1/2 \geq 0$

but minimum value of $\sin^2 x - \sin x + 1/2 = 1/4$

at $x = \pi/6$

$$\Rightarrow \sqrt{\sin^2 x - \sin x + \frac{1}{2}} \geq 1/2$$
$\because x \in [0, 3] \Rightarrow x = \pi/6, 5\pi/6$
$\therefore B = 2^{\sec^2 y} \leq 2$

because $\sec^2 y = \frac{1}{\cos^2 y}$

$\Rightarrow \quad |\cos x| \leq 1$

and $\sec^2 y \geq 1$

So, $B = 2$ and $A = 1/2$ is only possible solution

for $B = 2 \sec^2 y = 1$ in $y \in [0, 3]$

$\sec^2 y = 1$ at $y = 0$

therefore, possible pairs are

$(\pi/6, 0)$ and $(5\pi/6, 0)$

$\therefore$ Number of solutions are 2 .

Question3

The number of solutions of the equation $4 \cos 2\theta \cos 3\theta = \sec \theta$ in the interval $[0, 2\pi]$ is

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

12

B.

8

C.

16

D.

4

Answer: A

Solution:

Given,

$$4\cos 2\theta \cos 3\theta = \sec\theta$$

$\Rightarrow \quad 4\cos\theta\cos 2\theta \cdot \cos 3\theta = 1$

$\Rightarrow \quad 2\cos 2\theta(\cos 4\theta + \cos 2\theta) = 1$

$\Rightarrow \quad 2\cos 2\theta \cos 4\theta + 2\cos^2 2\theta = 1$

$\Rightarrow \quad 2\cos 2\theta \cos 4\theta + 2\cos^2 2\theta - 1 = 0$

$\Rightarrow \quad 2\cos 2\theta \cos 4\theta + \cos 4\theta = 0$

$\Rightarrow \quad \cos 4\theta(2\cos 2\theta + 1) = 0$

either $\quad \cos 4\theta = 0$

$\Rightarrow \quad 4\theta = (2n+1)\frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{8}$

$\Rightarrow \quad \cos 2\theta = -\frac{1}{2} \Rightarrow \cos 2\theta = \cos\frac{2\pi}{3}$

$\Rightarrow \quad 2\theta = 2n\pi \pm \frac{2\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

$\because \theta \in (0, 2\pi)$

$\therefore \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8} \rightarrow$ 8 solutions

and $\theta = \frac{\pi}{3}, \pi \pm \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \rightarrow$ 4 solutions

$\therefore$ Total number of solution $= 12$

Question4

$$\tan\frac{2\pi}{7} \cdot \tan\frac{4\pi}{7} + \tan\frac{4\pi}{7} \cdot \tan\frac{\pi}{7} + \tan\frac{\pi}{7} \cdot \tan\frac{2\pi}{7} =$$

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

7

B.

-7

C.

3

D.

-3

Answer: B

Solution:

Given, expression is

$\tan\left(\frac{2\pi}{7}\right) \cdot \tan\left(\frac{4\pi}{7}\right) + \tan\left(\frac{4\pi}{7}\right) \cdot \tan\left(\frac{\pi}{7}\right) + \tan\left(\frac{\pi}{7}\right) \cdot \tan\left(\frac{2\pi}{7}\right)$

Let $x = \frac{\pi}{7}, y = \frac{2\pi}{7}$ and $z = \frac{4\pi}{7}$

So, the expression becomes

$\tan(y) \cdot \tan(z) + \tan(z) \cdot \tan(x) + \tan(x) \cdot \tan(y)$

Now, $x + y + z = \frac{\pi}{7} + \frac{2\pi}{7} + \frac{4\pi}{7} = \frac{7\pi}{7} = \pi$

Since, $x + y + z = \pi$, we can use the identify

$\tan(x) \cdot \tan(y) + \tan(y) \cdot \tan(z) + \tan(z) \cdot \tan(x) = -7$

$\therefore \tan\left(\frac{\pi}{7}\right) \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{4\pi}{7}\right) \cdot \tan\left(\frac{2\pi}{7}\right) + \tan\left(\frac{4\pi}{7}\right) \tan\left(\frac{\pi}{7}\right) = -7$

Question5

The sum of the solutions of $\cos x \sqrt{16 \sin^2 x} = 1$ in $(0, 2\pi)$ is

AP EAPCET 2025 - 24th May Morning Shift

Options:

A.

2π

B.

$\frac{13\pi}{2}$

C.

$\frac{17\pi}{4}$

D.

4π

Answer: A

Solution:

Given, $\cos x \sqrt{16 \sin^2 x} = 1$

Since, $16 \sin^2 x = 4|\sin x|$

$$\begin{aligned} &\text{So, } \cos x(4|\sin x|) = 1 \\ &\Rightarrow \quad 2 \sin 2x = \pm 1 \end{aligned}$$

Also, $\sin 2x = \pm\frac{1}{2}$

$$\begin{aligned} x &= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12} \\ \therefore \quad \text{Sum } &= \frac{\pi}{12} + \frac{5\pi}{12} + \frac{7\pi}{12} + \frac{11\pi}{12} = 2\pi \end{aligned}$$

Question6

If $\sqrt{3} \cos\theta + \sin\theta > 0$, then

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Options:

A.

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

B.

$-\frac{\pi}{3} < \theta < \frac{2\pi}{3}$

C.

$-\frac{2\pi}{3} < \theta < \frac{\pi}{3}$

D.

$-\frac{\pi}{6} < \theta < \frac{5\pi}{6}$

Answer: B

Solution:

$$\sqrt{3}\cos\theta + \sin\theta > 0$$
$$\Rightarrow \quad \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta > 0$$
$$\Rightarrow \quad \cos\frac{\pi}{6}\cos\theta + \sin\frac{\pi}{6}\sin\theta > 0$$
$$\Rightarrow \quad \cos\left(\theta - \frac{\pi}{6}\right) > 0$$
$$\because \quad \cos x > 0$$
$$\Rightarrow \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
$$\therefore \quad \frac{-\pi}{2} < \theta - \frac{\pi}{6} < \frac{\pi}{2}$$
$$\Rightarrow \quad -\frac{\pi}{2} + \frac{\pi}{6} < \theta < \frac{\pi}{2} + \frac{\pi}{6}$$
$$\Rightarrow \quad -\frac{\pi}{3} < \theta < \frac{2\pi}{3}$$

Hence, $\frac{-\pi}{3} < \theta < \frac{2\pi}{3}$

Question7

The general solution satisfying both the equations $\sin x = -\frac{3}{5}$ and $\cos x = -\frac{4}{5}$ is

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Options:

A.

$x = (2n+1)\pi + \tan^{-1}\left(\frac{3}{4}\right), n \in Z$

B.

$x = 2n\pi + \tan^{-1}\left(\frac{3}{4}\right), n \in Z$

C.

$x = n\pi + \tan^{-1}\left(\frac{3}{4}\right), n \in Z$

D.

$x = n\pi \pm \tan^{-1}\left(\frac{3}{4}\right), n \in Z$

Answer: A

Solution:

$\because \sin x = -3/5$ and $\cos x = -4/5$

$\sin x$ and $\cos x$ both are negative

So, x must be lie in 3rd quadrant

Therefore,

$$x = (2n+1)\pi + \theta$$
$$\tan\theta = \frac{(-3/5)}{(-4/5)} = 3/4$$
$$\Rightarrow \quad \theta = \tan^{-1}(3/4)$$

Hence, $x = (2n+1)\pi + \tan^{-1}(3/4), n \in z$

Question8

The number of solutions of the equation $\sec x \cdot \cos 5x + 1 = 0$ in the interval $[0, 2\pi]$ is

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Options:

A.

5

B.

8

C.

10

D.

12

Answer: C

Solution:

Given, trigonometric equation is

$\sec x \cdot \cos 5x + 1 = 0$

$\Rightarrow \dfrac{\cos 5x + \cos x}{\cos x} = 0$

$\Rightarrow \quad 2 \cdot \cos 3x \cdot \cos 2x = 0$

$\Rightarrow \quad \cos 3x = 0 \text{ or } \cos 2x = 0$

$\Rightarrow \quad 3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

$\left(\begin{smallmatrix} \because 0 \leq x \leq 2\pi \\ \therefore 0 \leq 3x \leq 6\pi \end{smallmatrix}\right)$

$\Rightarrow \quad x = \dfrac{\pi}{6}, \dfrac{3\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{9\pi}{6}, \dfrac{11\pi}{6}$

Also, $2x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2} \left(\begin{array}{l} \because 0 \leq x \leq 2\pi \\ \therefore 0 \leq 2x \leq 4\pi \end{array}\right)$

$\Rightarrow \quad x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

$\therefore$ Total number of solutions $= 10$

Question9

If $2 \sin x - \cos 2x = 1$, then $\left(3 - 2 \sin^2 x\right) =$

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Options:

A.

$\sqrt{3}$

B.

$-\sqrt{3}$

C.

$\sqrt{5}$

D.

$-\sqrt{5}$

Answer: C

Solution:

$$\begin{aligned}
& 2\sin x-\cos 2x=1\\
\Rightarrow & 2\sin x-\left(1-2\sin^2 x\right)=1\\
\Rightarrow & 2\sin x-1+2\sin^2 x=1\\
\Rightarrow & 2\sin^2 x+2\sin x-2=0\\
\Rightarrow & \sin^2 x+\sin x-1=0\\
\Rightarrow & \sin x=\frac{-1\pm\sqrt{1+4}}{2\times 1}\\
\Rightarrow & \sin x=\frac{-1\pm\sqrt{5}}{2}\\
\Rightarrow & \sin x=\frac{\sqrt{5}-1}{2}\\
\Rightarrow & 3-2\sin^2 x=3-2\frac{(\sqrt{5}-1)^2}{4}\\
& =\frac{12-2(5+1-2\sqrt{5})}{4}\\
& =\frac{12-12+4\sqrt{5}}{4}=\sqrt{5}
\end{aligned}$$

Question10

If $x \neq (2n+1)\frac{\pi}{4}$, then the general solutions of $\cos x+\cos 3x=\sin x+\sin 3x$ is

AP EAPCET 2025 - 22nd May Evening Shift

Options:

A.

$n\pi+\frac{\pi}{8}$

B.

$n\pi\pm\frac{\pi}{8}$

C.

$\frac{n\pi}{2}\pm\frac{\pi}{8}$

D.

$\frac{n\pi}{2}+\frac{\pi}{8}$

Answer: D

Solution:

$$\begin{aligned}
&\cos x+\cos 3x=\sin x+\sin 3x \\
&\Rightarrow 2\cos 2x\cdot\cos x=2\sin 2x\cdot\cos x \\
&\Rightarrow \cos x(\cos 2x-\sin 2x)=0 \\
&\text{but given } x\neq(2n+1)\frac{\pi}{4} \\
&\Rightarrow \quad \cos 2x-\sin 2x=0 \\
&\Rightarrow \quad \sin 2x=\cos 2x \\
&\Rightarrow \quad \tan 2x=1 \\
&\Rightarrow \quad \tan 2x=\tan\frac{\pi}{4} \\
&\therefore \quad 2x=n\pi+\frac{\pi}{4}\Rightarrow x=\frac{n\pi}{2}+\frac{\pi}{8}
\end{aligned}$$

Question11

The number of solutions of $\sin 2x+\cos 4x=2$ in the interval $[-\pi,\pi]$ is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

3

B.

2

C.

0

D.

1

Answer: C

Solution:

$\sin 2x + \cos 4x = 2x \in [-\pi, \pi]$

$$\begin{aligned} \Rightarrow \quad & \sin 2x + 1 - 2\sin^2 2x = 2 \\ \Rightarrow \quad & 2\sin^2 2x - \sin 2x + 1 = 0 \\ \Rightarrow \quad & \sin 2x = \frac{1 \pm \sqrt{1-8}}{2} \end{aligned}$$

Not possible

Number of solution $= 0$

Question12

Number of solutions of the equation $\cos\theta + \cos 2\theta - \sqrt{3}(\sin\theta + \sin 2\theta) + 1 = 0$ lying in the interval $(0, 2\pi)$ is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

3

B.

6

C.

5

D.

4

Answer: D

Solution:

We have,

$$\cos\theta + \cos 2\theta - \sqrt{3}(\sin\theta + \sin 2\theta) + 1 = 0$$

$$\Rightarrow \quad \cos\theta + 2\cos^2\theta - \sqrt{3}\sin\theta - 2\sqrt{3}\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad (\cos\theta - \sqrt{3}\sin\theta) + \left(2\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta\right) = 0$$

$$\Rightarrow 1(\cos\theta - \sqrt{3}\sin\theta) + 2\cos\theta(\cos\theta - \sqrt{3}\sin\theta) = 0$$

$$\Rightarrow \quad (\cos\theta - \sqrt{3}\sin\theta)(2\cos\theta + 1) = 0$$

$$\Rightarrow \cos\theta - \sqrt{3}\sin\theta = 0 \text{ or } 2\cos\theta = -1$$

$$\tan\theta = \frac{1}{\sqrt{3}} \text{ or } \cos\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$\theta = n\pi + \frac{\pi}{6} \quad \text{or } \theta = 2n\pi + \frac{2\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{7\pi}{6} \quad \text{or } \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$\because$ Total number of solution in the interval $(0, 2\pi)$ is 4 .

Question13

The number of solutions of the equation $2\sin^2\theta - 3\cos^2\theta = \sin\theta\cos\theta$ lying in the intervals $(-\pi, \pi)$ is

AP EAPCET 2025 - 21st May Morning Shift

Options:

A.

2

B.

4

C.

3

D.

1

Answer: B

Solution:

We have,

$2\sin^2\theta - 3\cos^2\theta = \sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$, we get $\cos\theta \neq 0$

$$\begin{aligned} \Rightarrow \quad & 2\tan^2\theta - 3 = \tan\theta \\ \Rightarrow \quad & 2\tan^2\theta - \tan\theta - 3 = 0 \\ \Rightarrow \quad & (2\tan\theta - 3)(\tan\theta + 1) = 0 \\ \because \quad & \tan\theta = \frac{3}{2} \text{ and } \tan\theta = -1 \end{aligned}$$

Clearly, in the interval $(-\pi, \pi)$ $\tan\theta = \frac{3}{2}$ gives two solution and $\tan\theta = -1$ gives two solution $\therefore$ Total number of solution $= 4$

Question14

The general solution of $\cot\frac{x}{2} - \cot x = \operatorname{cosec}\frac{x}{2}$ is

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Options:

A. $\left\{2n\pi + \frac{\pi}{3} \middle| n \in z\right\}$

B. $\left\{4n\pi + \frac{\pi}{3} \middle| n \in z\right\}$

C. $\left\{2n\pi + \frac{2\pi}{3} \middle| n \in z\right\}$

D. $\left\{4n\pi \pm \frac{2\pi}{3} \middle| n \in z\right\}$

Answer: D

Solution:

Given the equation:

$\cot\frac{x}{2} - \cot x = \operatorname{cosec}\frac{x}{2}$

We can express this equation using trigonometric identities:

$\Rightarrow \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{1}{\sin x} = \operatorname{cosec}\frac{x}{2}$

$\Rightarrow \left(\frac{\cos\frac{x}{2}-1}{\sin\frac{x}{2}}\right) = \frac{\cos x}{\sin x}$

$\Rightarrow \frac{\cos\frac{x}{2}-1}{\sin\frac{x}{2}} = \frac{\cos x}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$\Rightarrow 2(\cos\frac{x}{2}-1) = \frac{\cos x}{\cos\frac{x}{2}}$

$\Rightarrow 2\cos^2\frac{x}{2} - 2\cos\frac{x}{2} = \cos x$

Using the identity $\cos x = 2\cos^2\frac{x}{2} - 1$:

$\Rightarrow 2\cos^2\frac{x}{2} - 2\cos\frac{x}{2} = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow \cos\frac{x}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$

This implies:

$\Rightarrow \frac{x}{2} = 2n\pi \pm \frac{\pi}{3}$

Thus, we find the general solution for x:

$\Rightarrow x = 4n\pi \pm \frac{2\pi}{3}$

Question15

The values of x in $(-\pi, \pi)$, which satisfy the equation $8^{1+\cos^2 x+\cos^4 x+\ldots\ldots} = 4^3$ are

AP EAPCET 2024 - 22th May Evening Shift

Options:

A. $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

B. $\pm\frac{\pi}{6}, \frac{\pi}{3}$

C. $\pm\frac{\pi}{8}$

D. $\frac{\pi}{3}$

Answer: A

Solution:

Given that

$$8^{1+\cos^2 x+\cos^4 x+\ldots} = 4^3$$

$\Rightarrow \quad 2^{3\left(1+\cos^2 x+\cos^4 x+\ldots\right)} = 2^6$

$\Rightarrow \quad 3\left(1+\cos^2 x+\cos^4 x+\ldots.\right) = 6$

$\Rightarrow \quad 1+\cos^2 x+\cos^4 x+\ldots = 2$

$\Rightarrow \quad \cos^2 x+\cos^4 x+\ldots. = 1$

$\because \quad$ Sum of GP of infinite terms $= \frac{a}{1-r}$

$\therefore \quad \frac{\cos^2 x}{1-\cos^2 x} = 1 \Rightarrow \cos^2 x = 1-\cos^2 x$

$\Rightarrow \quad 2\cos^2 x = 1 \Rightarrow \cos^2 x = \frac{1}{2}$

$\Rightarrow \quad x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

Question16

The general solution of the equation $\tan x+\tan 2x-\tan 3x=0$ is

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. $\left\{x \mid x=n\pi\pm\frac{\pi}{3} \text{ or } \frac{n\pi}{2}, n\in Z\right\}$

B. $\left\{x \mid x=n\pi\pm\frac{\pi}{3} \text{ or } n\pi, n\in Z\right\}$

C. $\left\{x \mid x=n\pi\pm\frac{\pi}{3} \text{ or } \frac{n\pi}{2} \text{ or } n\pi, n\in Z\right\}$

D. $\left\{x \mid x=n\pi\pm\frac{\pi}{6} \text{ or } \frac{n\pi}{2}, n\in Z\right\}$

Answer: B

Solution:

To solve the equation $\tan x+\tan 2x-\tan 3x=0$, we start by understanding the identity given:

$\tan x+\tan 2x=\tan 3x$.

Using trigonometric identities, we have:

$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

$\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$.

Substituting these into the equation gives:

$\tan x+\frac{2\tan x}{1-\tan^2 x}=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$.

Simplifying this, we obtain:

$\frac{\tan x(3-\tan^2 x)}{1-\tan^2 x} = \frac{\tan x(3-\tan^2 x)}{1-3\tan^2 x}$.

Clearing the denominators by multiplying through by $(1-\tan^2 x)(1-3\tan^2 x)$, we get:

$\tan x(3-\tan^2 x)(1-3\tan^2 x) = \tan x(3-\tan^2 x)(1-\tan^2 x)$.

Subtracting gives:

$\tan x(3-\tan^2 x)\{-2\tan^2 x\} = 0$.

This implies:

$\tan^3 x = 0$ or

$3-\tan^2 x = 0$.

Solving each equation separately:

$\tan^3 x = 0 \implies \tan x = 0 \implies x = n\pi,\, n \in \mathbb{Z}$.

$3-\tan^2 x = 0 \implies \tan^2 x = 3 \implies \tan x = \pm\sqrt{3}$.

Solving $\tan x = \pm\sqrt{3}$, we get:

$\tan x = \sqrt{3} \implies x = n\pi + \frac{\pi}{3},\, n \in \mathbb{Z}$.

$\tan x = -\sqrt{3} \implies x = n\pi - \frac{\pi}{3},\, n \in \mathbb{Z}$.

Therefore, the general solution is:

$\{x \mid x = n\pi \pm \frac{\pi}{3},\, n\pi,\, n \in \mathbb{Z}\}$.

Question17

Number of solutions of the trigonometric equation $2\tan 2\theta - \cot 2\theta + 1 = 0$ lying in the interval $[0, \pi]$ is

AP EAPCET 2024 - 21th May Evening Shift

Options:

A. 2

B. 3

C. 4

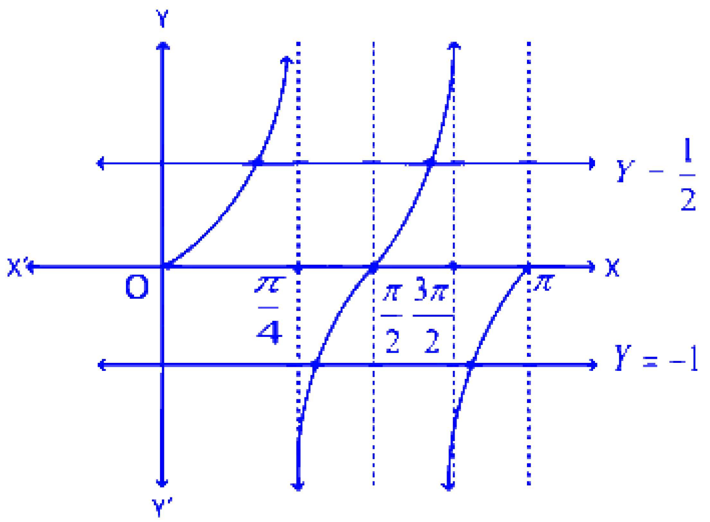
D. 5

Answer: C

Solution:

Given, $2 \tan 2\theta - \cot 2\theta + 1 = 0, \theta \in [0, x]$

$$\Rightarrow \quad 2 \tan 2\theta - \frac{1}{\tan 2\theta} + 1 = 0$$
$$\Rightarrow \quad 2 \tan^2 2\theta + \tan 2\theta - 1 = 0$$
$$\Rightarrow \quad 2 \tan^2 2\theta + 2 \tan 2\theta - \tan 2\theta - 1 = 0$$
$$\Rightarrow \quad 2 \tan 2\theta(1 + \tan 2\theta) - (1 + \tan 2\theta) = 0$$
$$\Rightarrow \quad (2 \tan 2\theta - 1)(1 + \tan 2\theta) = 0$$
$$\Rightarrow \quad 2 \tan 2\theta = 1 \text{ or } \tan 2\theta = -1$$
$$\Rightarrow \quad \tan 2\theta = \frac{1}{2}$$
$$\text{or} \quad \tan 2\theta = \tan\left(n\pi + \frac{3\pi}{4}\right)$$
$$\Rightarrow \quad 2\theta = n\pi + \frac{3\pi}{4} \Rightarrow \theta = \frac{n\pi}{2} + \frac{3\pi}{8}$$



Since, number of point of intersection is 4 .

Hence, number of solution is 4 .

Question18

For $a \in R - \{0\}$, if $a \cos x + a \sin x + a = 2k + 1$ has a solution, then k lies in the interval

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $\left[\frac{a-1-\sqrt{2}a}{2}, \frac{a-1+\sqrt{2}a}{2}\right]$

B. $\left[\frac{a+1-\sqrt{2}}{2}, \frac{a+1+\sqrt{2}}{2}\right]$

C. $\left[\frac{a-1-\sqrt{2}}{2}, \frac{a-1+\sqrt{2}}{2}\right]$

D. $\left[-\frac{\left(\sqrt{2a^2+2a+1}+1\right)}{2}, \frac{\left(\sqrt{2a^2+2a+1}-1\right)}{2}\right]$

Answer: A

Solution:

Given the equation:

$a \cos x + a \sin x + a = 2k + 1$

we can rearrange it as:

$a(\cos x + \sin x) = 2k + 1 - a$

Divide through by a (since $a \neq 0$):

$\cos x + \sin x = \frac{2k+1-a}{a} = \frac{2k+1}{a} - 1$

Recognizing that:

$\cos x + \sin x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$

Let $\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \sin\left(x + \frac{\pi}{4}\right)$, then:

$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = \frac{2k+1}{a} - 1$

So:

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\left(\frac{2k+1-a}{a}\right)$

For $\sin(y)$ where y is in the range $[-1, 1]$:

$-1 \leq \frac{2k+1-a}{\sqrt{2}a} \leq 1$

This implies:

$-a\sqrt{2} \leq 2k + 1 - a \leq a\sqrt{2}$

Rearranging gives:

$a - a\sqrt{2} \leq 2k + 1 \leq a\sqrt{2} + a$

Subtract 1 from each part:

$a - a\sqrt{2} - 1 \leq 2k \leq a\sqrt{2} + a - 1$

Finally, divide by 2:

$\frac{a - a\sqrt{2} - 1}{2} \leq k \leq \frac{a\sqrt{2} + a - 1}{2}$

Thus, k lies in the interval:

$\left[\frac{a - \sqrt{2}a - 1}{2}, \frac{a + \sqrt{2}a - 1}{2}\right]$

Therefore, k is within the interval $\left[\frac{a - 1 - \sqrt{2}a}{2}, \frac{a - 1 + \sqrt{2}a}{2}\right]$.

Question19

If the general solution .set of $\sin x + 3\sin 3x + \sin 5x = 1$ is S, then $\{\sin \alpha / \alpha \in S\} =$

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $\{1, -1, 0\}$

B. $\left\{\frac{1}{2}, \frac{-1}{2}, 0, 1, -1\right\}$

C. $\left\{\frac{\sqrt{3}}{2}, 0, \frac{-\sqrt{3}}{2}\right\}$

D. $\left\{1, -1, \frac{\sqrt{3}}{2}, 0, \frac{-\sqrt{3}}{2}\right\}$

Answer: C

Solution:

To solve the equation $\sin x + 3\sin 3x + \sin 5x = 1$, we start by rewriting it as:

$(\sin x + \sin 5x) + 3\sin 3x = 0$

Using the sum-to-product identities, the expression $\sin x + \sin 5x$ becomes:

$2\sin 3x \cos 2x$

Thus, the equation becomes:

$2\sin 3x \cos 2x + 3\sin 3x = 0$

Factoring out $\sin 3x$, we get:

$\sin 3x(2\cos 2x + 3) = 0$

Which implies that:

$\sin 3x = 0$

$2\cos 2x + 3 = 0$

For $\sin 3x = 0$, the solutions are:

$3x = n\pi \quad \Rightarrow \quad x = \frac{n\pi}{3}, \quad n \in \mathbb{Z}$

Next, for $2\cos 2x + 3 = 0$, solving gives:

$\cos 2x = -\frac{3}{2}$

However, since the cosine of an angle cannot exceed 1 or be less than -1, there are no solutions in real numbers for this equation.

Since the only valid solutions come from $\sin 3x = 0$, we consider the possible values of $\sin \alpha$ where $\alpha \in S$. Evaluating sin at these angles, we find:

$\{\sin \alpha \mid \alpha \in S\} = \left\{\frac{\sqrt{3}}{2}, 0, \frac{-\sqrt{3}}{2}\right\}$

Question20

The general solution of the equation $\sin^2 \theta + 3\cos^2 \theta = 5\sin \theta$ is

AP EAPCET 2024 - 20th May Evening Shift

Options:

A. $n\pi \pm \frac{\pi}{3}, n \in Z$

B. $n\pi + (-1)^n \frac{\pi}{6}, n \in Z$

C. $n\pi \pm \frac{\pi}{6}, n \in Z$

D. $n\pi + (-1)^n \frac{\pi}{3}, n \in Z$

Answer: B

Solution:

Given, $\sin^2 \theta + 3\cos^2 \theta = 5\sin \theta$

$$\Rightarrow \quad \sin^2\theta + 3\left(1-\sin^2\theta\right) = 5\sin\theta$$
$$\Rightarrow \quad \sin^2\theta + 3 - 3\sin^2\theta = 5\sin\theta$$
$$\Rightarrow \quad -2\sin^2\theta - 5\sin\theta + 3 = 0$$
$$\Rightarrow \quad 2\sin^2\theta + 5\sin\theta - 3 = 0$$
$$\Rightarrow \quad \sin\theta = \frac{-6}{2}, \sin\theta = \frac{1}{2}$$
$$\therefore \quad \sin\theta = \frac{1}{2} \quad [\because |\sin\theta| \leq 1]$$
$$\Rightarrow \quad \theta = n\pi + (-1)^n\frac{\pi}{6}$$

$\therefore$ The general solution is $n\pi + (-1)^n\frac{\pi}{6}, n \in Z$.

Question21

The number of ordered pairs (x, y) satisfying the equations $\sin x + \sin y = \sin(x+y)$ and $|x| + |y| = 1$ is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. 2

B. 3

C. 4

D. 6

Answer: D

Solution:

We are given two equations: $\sin x + \sin y = \sin(x+y)$ and $|x| + |y| = 1$.

Let's analyze the first equation:

$$\sin x + \sin y = \sin(x+y)$$
$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right) = 2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x+y}{2}\right)$$
$$\Rightarrow \sin\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\right] = 0$$
$$\Rightarrow \sin\left(\frac{x+y}{2}\right)\left[-2\sin\frac{x}{2}\cdot\sin\left(-\frac{y}{2}\right)\right] = 0$$
$$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\sin\frac{x}{2}\cdot\sin\frac{y}{2} = 0$$

From this, we have three cases to consider:

Case 1: $\sin\left(\frac{x+y}{2}\right) = 0$

This implies:

$\frac{x+y}{2} = 0 \Rightarrow x + y = 0 \Rightarrow x = -y$

Using the second equation $|x| + |y| = 1$:

$|x| + |-x| = 1 \Rightarrow x = \pm\frac{1}{2} \quad \text{and} \quad y = \mp\frac{1}{2}$

Thus, the ordered pairs are $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

Case 2: $\sin\frac{x}{2} = 0$

This gives:

$\frac{x}{2} = 0 \Rightarrow x = 0$

From $|x| + |y| = 1$:

$0 + |y| = 1 \Rightarrow |y| = 1 \Rightarrow y = \pm 1$

So, the ordered pairs are $(0, 1)$ and $(0, -1)$.

Case 3: $\sin\frac{y}{2} = 0$

This implies:

$\frac{y}{2} = 0 \Rightarrow y = 0$

Using $|x| + |y| = 1$:

$|x| + 0 = 1 \Rightarrow |x| = 1 \Rightarrow x = \pm 1$

Therefore, the ordered pairs are $(1, 0)$ and $(-1, 0)$.

By combining all the cases, we have a total of 6 ordered pairs:

$\left(\frac{1}{2}, -\frac{1}{2}\right)$

$\left(-\frac{1}{2}, \frac{1}{2}\right)$

$(0, 1)$

$(0, -1)$

$(1, 0)$

$(-1, 0)$

Question22

If $5 \sin hx - \cos hx = 5$, then one of the values of $\tan hx$ is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. $\frac{2}{5}$

B. $\frac{3}{5}$

C. $\frac{-3}{5}$

D. $\frac{-1}{5}$

Answer: C

Solution:

We have, $5 \sinh(x) - \cosh(x) = 5$

$$\Rightarrow \quad 5\left(\frac{e^x - e^{-x}}{2}\right) - \left(\frac{e^x + e^{-x}}{2}\right) = 5$$

$$\Rightarrow \quad \frac{5e^x - 5e^{-x} - e^x - e^{-x}}{2} = 5$$

$$\Rightarrow \quad \frac{4e^x - 6e^{-x}}{2} = 5 \Rightarrow 2e^x - 3e^{-x} = 5$$

$$\Rightarrow \quad 2e^{2x} - 3 = 5e^x \Rightarrow 2e^{2x} - 5e^x - 3 = 0$$

Let, $e^x = y$

Then, $2y^2 - 5y - 3 = 0$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm \sqrt{49}}{4} = \frac{5 \pm 7}{4}$$

$$= \frac{5 + 4}{4}, \frac{5 - 7}{4} = 3, -\frac{1}{2}$$

Thus, $e^x = 3$ or $e^x = -\frac{1}{2}$

If $e^x = 3$

Then,

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{3 - \frac{1}{3}}{3 + \frac{1}{3}}$$
$$= \frac{8/3}{10/3} = \frac{8}{10} = \frac{4}{5}$$

If $e^x = -\frac{1}{2}$

$$Then, \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$
$$= \frac{-\frac{1}{2} + 2}{-\frac{1}{2} - 2} = \frac{\frac{3}{2}}{-\frac{5}{2}} = -\frac{3}{5}$$

Question23

The smallest positive value (in degrees) of θ for which $\tan(\theta + 100^\circ) = \tan(\theta + 50^\circ)\tan(\theta)\tan(\theta - 50^\circ)$ is valid, is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. 60°

B. 45°

C. 30°

D. 15°

Answer: C

Solution:

We need to find the smallest positive value of θ for which the equation

$\tan(\theta + 100^\circ) = \tan(\theta + 50^\circ)\tan(\theta)\tan(\theta - 50^\circ)$

holds true.

Step-by-Step Solution:

Start with the given equation:

$\tan(\theta + 100^\circ) = \tan(\theta + 50^\circ)\tan(\theta)\tan(\theta - 50^\circ)$

Rearranging terms, we have:

$\frac{\tan(\theta+100^\circ)}{\tan(\theta-50^\circ)} = \tan(\theta+50^\circ)\tan(\theta)$

Express tangent in terms of sine and cosine:

$$\frac{\sin(\theta+50^\circ)}{\cos(\theta+100^\circ)\cos(\theta-50^\circ)} = \frac{\sin(\theta+50^\circ)\sin(\theta-50^\circ)}{\cos(\theta+50^\circ)\cos(\theta)}$$

Using the componendo and dividendo rule:

$$\frac{\sin(2\theta+50^\circ)}{\sin(150^\circ)} = \frac{-\cos 50^\circ}{\cos(2\theta+50^\circ)}$$

Equating the terms, we proceed with the identities:

$\sin(2\theta+50^\circ)\cos(2\theta+50^\circ) = -\cos 50^\circ \sin 150^\circ$

Multiply both sides by 2:

$\sin(4\theta+100^\circ) = -\sin 40^\circ$

Since $\sin(-x) = -\sin(x)$, we have:

$\sin(4\theta+100^\circ) = \sin(-40^\circ)$

The general solution for the sine equation is:

$4\theta + 100^\circ = n \cdot 180^\circ + (-1)^n \cdot (-40^\circ)$

For $n = 1$, substitute:

$4\theta + 100^\circ = 180^\circ + 40^\circ$

$4\theta + 100^\circ = 220^\circ$

Solve for θ:

$4\theta = 220^\circ - 100^\circ$

$4\theta = 120^\circ$

$\theta = 30^\circ$

Thus, the smallest positive value of θ is $\boxed{30^\circ}$.

Question24

The general solution of

$$4\cos 2x - 4\sqrt{3}\sin 2x + \cos 3x - \sqrt{3}\sin 3x + \cos x - \sqrt{3}\sin x = 0$$

Options:

A. $\frac{n\pi}{2}-\frac{\pi}{3}$

B. $\frac{n\pi}{2}+\frac{\pi}{6}$

C. $\frac{n\pi}{2}+\frac{\pi}{12}$

D. $\frac{n\pi}{2}-\frac{\pi}{12}$

Answer: C

Solution:

$4\cos 2x-4\sqrt{3}\sin 2x+\cos 3x$

$-\sqrt{3}\sin 3x+\cos x-\sqrt{3}\sin x=0$

Divide by 2 on both sides, we get

$$\left[\tfrac{1}{2}\cos 2x-\tfrac{\sqrt{3}}{2}\sin 2x\right]$$
$$+\left[\tfrac{1}{2}\cos 3x-\tfrac{\sqrt{3}}{2}\sin 3x\right]+\left[\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x\right]=0\Rightarrow\left[\sin\frac{\pi}{6}\cos 2x-\cos\frac{\pi}{6}\sin 2x\right]$$
$$+$$

$$+\left[\sin\frac{\pi}{6}\cos 3x-\cos\frac{\pi}{6}\sin 3x\right]$$
$$+\left[\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right]=0$$
$$\Rightarrow 4\sin\left(\frac{\pi}{6}-2x\right)+\sin\left(\frac{\pi}{6}-3x\right)$$
$$+\sin\left(\frac{\pi}{6}-x\right)=0$$
$$\Rightarrow 4\sin\left(\frac{\pi}{6}-2x\right)+2\sin\left(\frac{\pi}{6}-2x\right)$$
$$\Rightarrow$$
$$\Rightarrow 2\sin\left(\frac{\pi}{6}-2x\right)[\cos x+2]=0$$
$$\Rightarrow 2\sin\left(\frac{\pi}{6}-2x\right)=0$$

$$\frac{\pi}{6}-2x=-n\pi$$

$$2x=n\pi+\frac{\cos x\neq -2]}{6}\Rightarrow x=\frac{n\pi}{2}+\frac{\pi}{12}$$

Question25

The general solution of $2\cos^2 x - 2\tan x + 1 = 0$ is

AP EAPCET 2024 - 19th May Evening Shift

Options:

A. $n\pi + \frac{\pi}{4}, n \in Z$

B. $2n\pi \pm \frac{\pi}{4}, n \in Z$

C. $2n\pi \pm \frac{\pi}{3}, n \in Z$

D. $n\pi \pm \frac{\pi}{3}, n \in Z$

Answer: A

Solution:

To solve the equation $2\cos^2 x - 2\tan x + 1 = 0$, we proceed with the following steps:

Start by rewriting the equation:

$2\cos^2 x - 2\tan x + 1 = 0$

Using the identity $\cos^2 x = \frac{1+\cos 2x}{2}$, substitute:

$2\left(\frac{1+\cos 2x}{2}\right) - 2\tan x + 1 = 0$

This simplifies to:

$\cos 2x - 2\tan x + 2 = 0$

Express $\cos 2x$ in terms of $\tan x$:

$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

Substitute this identity:

$\frac{1-\tan^2 x}{1+\tan^2 x} - 2\tan x + 2 = 0$

Eliminate the fraction by multiplying through by $1 + \tan^2 x$:

$1 - \tan^2 x - 2\tan x + 2\tan^2 x + 2 = 0$

Simplify the equation:

$-2\tan^3 x + \tan^2 x - 2\tan x + 3 = 0$

Rearrange and factor the polynomial, letting $\tan x = y$:

$2y^3 - y^2 + 2y - 3 = 0$

Factoring, we get:

$(y-1)(2y^2 + y + 3) = 0$

Solve the factors:

$y - 1 = 0 \quad \Rightarrow \quad y = 1$

The quadratic $2y^2 + y + 3$ does not have real solutions (since its discriminant is negative).

Therefore, $\tan x = 1$, which implies:

$x = n\pi + \frac{\pi}{4}, \quad n \in \mathbb{Z}$

The general solution to the equation is thus $x = n\pi + \frac{\pi}{4}$ where n is any integer.

Question26

$1 + \sin x + \sin^2 x + \sin^3 x + \cdots + \infty = 4 + 2\sqrt{3}$ and $0 < x < \pi, x \neq \frac{\pi}{2}$ then $x =$

AP EAPCET 2024 - 18th May Morning Shift

Options:

A.

$\frac{\pi}{6}, \frac{\pi}{4}$

B.

$\frac{\pi}{4}, \frac{5\pi}{6}$

C.

$\frac{2\pi}{5}, \frac{\pi}{6}$

D.

$\frac{\pi}{3}, \frac{2\pi}{3}$

Answer: D

Solution:

To solve the given series equation:

$1 + \sin x + \sin^2 x + \sin^3 x + \ldots = 4 + 2\sqrt{3}$

we recognize that this is an infinite geometric series with the first term $a = 1$ and the common ratio $r = \sin x$. The sum of an infinite geometric series is given by:

$S = \frac{a}{1-r}$

Here, substituting the given values:

$\frac{1}{1-\sin x} = 4 + 2\sqrt{3}$

To find $\sin x$, we solve:

$1 - \sin x = \frac{1}{4+2\sqrt{3}}$

Rationalizing the denominator gives:

$1 - \sin x = \frac{4-2\sqrt{3}}{4} = 1 - \frac{\sqrt{3}}{2}$

Thus:

$\sin x = \frac{\sqrt{3}}{2}$

Given the constraints $0 < x < \pi$ and $x \neq \frac{\pi}{2}$, we find that x could be $\frac{\pi}{3}$ or $\frac{2\pi}{3}$. The solutions within this range are:

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

Question27

$$\text{If } \sin\theta + \operatorname{cosec}\theta = 4, \text{ then } \sin^2\theta + \operatorname{cosec}^2\theta =$$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. 12

B. 18

C. 16

D. 14

Answer: D

Solution:

We have, $\sin\theta + \operatorname{cosec}\theta = 4$

$\Rightarrow \frac{1}{\operatorname{cosec}\theta} + \operatorname{cosec}\theta = 4$

Squaring both sides,

$$\frac{1}{\operatorname{cosec}^2\theta} + \operatorname{cosec}^2\theta + 2 = 16$$
$$\Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 16 - 2 = 14$$

Question28

If $2\cosh 2x + 10\sinh 2x = 5$, then $x =$

AP EAPCET 2022 - 5th July Morning Shift

Options:

A. $\frac{1}{2}\log\frac{4}{3}$

B. $\frac{1}{2}\log\frac{2}{3}$

C. $\frac{1}{2}\log\frac{3}{2}$

D. $\frac{1}{2}\log\frac{3}{4}$

Answer: A

Solution:

Here, $2\cosh 2x + 10\sinh 2x = 5$

We know that,

$$\cosh 2x = \frac{1}{2}\left(e^{2x} + e^{-2x}\right)$$
$$\sinh 2x = \frac{1}{2}\left(e^{2x} - e^{-2x}\right)$$

So, $2\left[\frac{1}{2}\left(e^{2x} + e^{-2x}\right)\right] + 10\left[\frac{1}{2}\left(e^{2x} - e^{-2x}\right)\right] = 5$

$$\Rightarrow e^{2x} + e^{-2x} + 5e^{2x} - 5e^{-2x} = 5$$
$$\Rightarrow 6e^{2x} - 5 - 4e^{-2x} = 0 \Rightarrow \left(3e^{2x} - 4\right)\left(2e^{2x} + 1\right) = 0$$
$$e^{2x} = \frac{4}{3} \text{ or } e^{2x} = -\frac{1}{2}$$

The only real solution occurs when $e^{2x} > 0$.

$$\text{So, } 2x = \ln\frac{4}{3}$$
$$\Rightarrow \quad x = \frac{1}{2}\ln\frac{4}{3}$$

Question29

If $\sin\left(\frac{\pi}{4}\cos\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$, then θ is equal to

AP EAPCET 2021 - 20th August Morning Shift

Options:

A. $2n\pi + \frac{\pi}{4}$

B. $2n\pi \pm \frac{\pi}{4}$

C. $2n\pi - \frac{\pi}{4}$

D. $n\pi + \frac{\pi}{4}$

Answer: D

Solution:

$$\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$$
$$\sin A = \cos B$$
$$\text{If } A + B = \frac{\pi}{2}$$
$$\Rightarrow \quad \sin A = \cos B$$
$$\Rightarrow \quad \frac{\pi}{4}(\tan\theta + \cot\theta) = \frac{\pi}{2}$$
$$\Rightarrow \quad \tan\theta + \frac{1}{\tan\theta} = 2 \Rightarrow \tan^2\theta \cdot 2\tan\theta + 1 = 0$$
$$\Rightarrow \quad (\tan\theta - 1)^2 = 0 \Rightarrow \tan^2\theta - 1 = 0$$
$$\Rightarrow \quad \tan\theta = 1$$
$$\theta = n\pi + \frac{\pi}{4}$$

Question30

If $\theta \in [0, 2\pi]$ and $\cos 2\theta = \cos\theta + \sin\theta$, then the sum of all values of θ satisfying the equation is

AP EAPCET 2021 - 19th August Evening Shift

Options:

A. $\frac{21\pi}{2}$

B. $\frac{11\pi}{4}$

C. $\frac{24\pi}{4}$

D. $\frac{31\pi}{4}$

Answer: C

Solution:

$\cos 2\theta = \cos\theta + \sin\theta$

$\Rightarrow \quad (\cos^2\theta - \sin^2\theta) - (\cos\theta + \sin\theta) = 0$

$\Rightarrow \quad (\cos\theta + \sin\theta)(\cos\theta - \sin\theta - 1) = 0$

$\Rightarrow \quad \cos\theta + \sin\theta = 0$

or $\cos\theta - \sin\theta = 1$

When $\cos\theta + \sin\theta = 0$, then $\tan\theta = -1$

then $\theta = n\pi + \frac{3\pi}{4} \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \in [0, 2\pi]$

and when $\cos\theta - \sin\theta = 1$, then $\sin\theta + (1 - \cos\theta) = 0$

and $\quad 2\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right) = 0$

and $\quad \sin\frac{\theta}{2} = 0, \cos\frac{\theta}{2} + \sin\frac{\theta}{2} = 0$

$\Rightarrow \quad \frac{\theta}{2} = n\pi, \tan\frac{\theta}{2} = -1$

$\Rightarrow \quad \theta = 0, 2\pi \in [0, 2\pi]$,

$\theta = \frac{6\pi}{4} \in [0, 2\pi]$

Sum of all values of θ

$= \frac{3\pi}{4} + \frac{7\pi}{4} + 0 + 2\pi + \frac{6\pi}{4} = \frac{24\pi}{4}$

Question31

The value of x satisfying the equation $3\operatorname{cosec} x = 4\sin x$ are

AP EAPCET 2021 - 19th August Morning Shift

Options:

A. $\frac{\pi}{6}, \frac{\pi}{3}$

B. $\pm\frac{\pi}{6}$

C. $\pm\frac{\pi}{3}$

D. $\frac{\pi}{3}, \frac{\pi}{4}$

Answer: C

Solution:

$3 \operatorname{cosec} x = 4 \sin x$

or $\sin^2 x = 3/4$

or $\sin^2 x = \sin^2 \frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$

or $x = \pm \frac{\pi}{3}$
